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Description of 2-local two sided multiplication on an algebra of matrices

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MATRITSALAR ALGEBRASIDA 2-LOKAL IKKI TOMONLAMA KO`PAYTIRISHLAR TAVSIFI

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Maqolada 2-lokal ikki tomonlama ko`paytirish tushunchasi kiritilgan va o`rganilgan. Har qanday barcha komponentalari musbat bo`lgan matritsalar yordamida aniqlangan Δ ikki lokal ikki tomonlama ko`paytirish uchun barcha $X \in M_2(R)$ matritsalarda $\Delta(X) = AXA$ tenglikni qanoatlantiruvchi $A \in M_2(R)$ matritsa mavjudligi isbotlangan. Shuningdek, agar ixtiyoriy barcha komponentalari musbat funksiya bo`lgan matritsalar yordamida aniqlangan $\Delta: M_2(R) \otimes C[a,b] \to M_2(R) \otimes C[a,b]$ ikki lokal ikki tomonlama ko`paytirish berilgan bo`lsa, u holda barcha $X \in M_2(R) \otimes C[a,b]$ matritsalar uchun $\Delta(X) = AXA$ tenglikni qanoatlantiruvchi $A \in M_2(R) \otimes C[a,b]$ matritsa mavjudligi isbotlangan.

***Kalit so`zlar:*** *Ikki o`lchamli matritsalar algebrasi, 2-lokal ikki tomonlama ko`paytirish, uzluksiz funksiyalar algebrasi.*

В данной работе введено и исследовано понятие 2-локального двустороннего умножения. Доказано что, для всякого 2-локального двустороного умножения Δ, определенного матрицами с положительными компонентами существует матрица $A \in M_2(R)$ такая, что $\Delta(X) = AXA$ для всех $X \in M_2(R)$. А также доказано, что для всякого 2-локального двустороного умножения $\Delta: M_2(R) \otimes C[a,b] \to M_2(R) \otimes C[a,b]$, определенного матрицами с положительными компонентами существует матрица $A \in M_2(R) \otimes C[a,b]$ такая, что $\Delta(X) = AXA$ для всех $X \in M_2(R) \otimes C[a,b]$.

Ключевые слова*: алгебра двумерных матриц, 2-локальное двустороное умножение, алгебра непрерывных функций.*

Kirish

Maqola matritsalar algebrasida ikki lokal ikki tomonlama ko`paytirish mavzusini o`rganishga bag`ishlangan. Bu tushuncha quyidagicha kiritiladi: $\Delta: \mathrm{M_2R} \to \mathrm{M_2R}$ akslantirish uchun ixtiyoriy $\mathrm{X}, \mathrm{Y} \in \mathrm{M_2(R)}$ olganda, shunday $\mathrm{A} \in \mathrm{M_2(R)}$ mavjud bo`lsaki, bunda $\Delta(\mathrm{x}) = \mathrm{AXA}$, $\Delta(\mathrm{y}) = \mathrm{AYA}$ tenglik bajarilsa, Δ ikki lokal ikki tomonlama ko`paytirish deyiladi.

1997-yilda P.Semrl [1] 2-lokal differensiallash tushunchasini kiritgan va cheksiz o`lchovli $H$ separabl Gilbert fazosi ustida aniqlangan barcha chegaralangan chiziqli operatorlarning $B(H)$ algebrasida har qanday 2-lokal differensiallash differensiallash bo`lishini ko`rsatgan. Keyinroq [2] maqolada chekli o`lchovli H Gilbert fazosida aniqlangan $B(H)$ algebra uchun ham shunday natija olingan [3]. Maqolada esa chekli o`lchovli butunlik halqalari ustida aniqlangan matritsalar halqasida har qanday 2-lokal differensiallash differensiallsh bo`lishi ko`rsatilgan. [4] Maqolada mualliflar yangi isbotlash usulini ishlab chiqib, Gilbert fazolari uchun yuqorida aytib o`tilgan [1] va [2] maqolalarning natijalarini umumlashtirishgan. Ya'ni, ular ixtiyoriy olingan (separabellik talab etilmaydi) H Gilbert fazosi ustida aniqlangan barcha chiziqli operatorlar $B(H)$ algebrasida 2-lokal differensiallashlarni o`rganishgan va $B(H)$ ustidagi har qanday 2-lokal differensiallash differensiallash bo`lishini isbotlashgan [5-11]. Maqolalarda mualliflar oldingi natijalarni kengaytirishgan va 2-lokal differensiallashlar bo`yicha bir qator natijalar olishgan.

Maqolada 2-lokal ikki tomonlama ko`paytitrish tushunchasi kiritilgan va o`rganilgan. Ikki o`lchovli matritsalar algebrasida barcha komponentalari musbat bo`lgan matritsalar yordamida aniqlangan Δ ikki lokal ikki tomonlama ko`paytirish uchun shunday $\mathrm{A} \in \mathrm{M_2(R)}$ matritsa mavjudligi isbotlanganki, bunda barcha birlik matritsalar $\Delta(\mathrm{e_{ij}}) = \mathrm{Ae_{ij}A}$ tenglikni qanoatlantiradi. Ushbu tasdiq maqolada lemma ko`rinishida bayon qilingan. Ushbu lemma yordamida, agar ixtiyoriy barcha komponentalari musbat bo`lgan matritsalar yordamida aniqlangan Δ ikki lokal ikki tomonlama ko`paytirish berilgan bo`lsa, u holda barcha $\mathrm{X} \in \mathrm{M_2(R)}$ matritsalar uchun $\Delta(\mathrm{X}) = \mathrm{AXA}$ tenglikni qanoatlantiruvchi $\mathrm{A} \in \mathrm{M_2(R)}$ matritsa mavjudligi haqidagi teorema isbotlangan. Shuningdek, bu teorema $\Delta: M_2(R) \otimes C[a,b] \to M_2(R) \otimes C[a,b]$ akslantirish uchun ham o`rinli bo`lishi isbotlangan, ya'ni, agar ixtiyoriy barcha komponentalari musbat bo`lgan matritsalar yordamida aniqlangan Δ ikki lokal ikki tomonlama ko`paytirish berilgan bo`lsa, u holda barcha $\mathrm{X} \in \mathrm{M_2(R)} \otimes C[a,b]$ matritsalar uchun $\Delta(\mathrm{X}) = \mathrm{AXA}$ tenglikni qanoatlantiruvchi $\mathrm{A} \in \mathrm{M_2(R)} \otimes C[a,b]$ matritsa mavjudligi isbotlangan.

2-lokal ikki tomonlama ko`paytirish

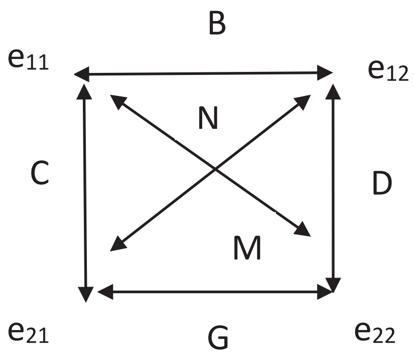
Ta'rif. Aytaylik, $\mathrm{M}_2(\mathrm{R})$ R haqiqiy sonlar maydoni ustida aniqlangan ikki o`lchovli matritsalar algebrasi bo`lsin. Agar $\Delta: \mathrm{M}_2(\mathrm{R}) \rightarrow \mathrm{M}_2(\mathrm{R})$ akslantirish uchun ixtiyoriy $\mathrm{x}, \mathrm{y} \in \mathrm{M}_2(\mathrm{R})$ matritsalarni olganda, shunday $\mathrm{A} \in \mathrm{M}_2(\mathrm{R})$ mavjud bo`lsaki, bunda $\Delta(\mathrm{x}) = \mathrm{AXA}$, $\Delta(\mathrm{y}) = \mathrm{AYA}$ tengliklar bajarilsa, u holda $\Delta$ *ikki lokal ikki tomonlama ko`paytirish* deyiladi.

Ushbu kiritilgan tushuncha bo`yicha quyidagi lemma o`rinlidir. Aytaylik, Δ bu $\mathrm{M}_2(\mathrm{R})$ ikki o`lchovli matritsalar algebrasida barcha komponentalari musbat bo`lgan matritsalar yordamida aniqlangan ikki lokal ikki tomonlama ko`paytirish bo`lsin.

Lemma 1. Ikki o`lchovli matritsalar algebrasida shunday $\mathrm{A} \in \mathrm{M}_2(\mathrm{R})$ matritsa mavjudki, barcha birlik $e_{ij} \in \mathrm{M}_2(\mathrm{R})$, $\mathrm{i}, \mathrm{j} = 1{,}2$ matritsalar uchun $\Delta$ikki lokal ikki tomonlama ko`paytirishda $\Delta(\mathrm{e}_{\mathrm{ij}}) = \mathrm{Ae}_{\mathrm{ij}}\mathrm{A}$ tenglik bajariladi, ya'ni:

$$\Delta(\mathrm{e}_{11}) = \mathrm{Ae}_{11}\mathrm{A},\ \Delta(\mathrm{e}_{12}) = \mathrm{Ae}_{12}\mathrm{A}$$
$$\Delta(\mathrm{e}_{21}) = \mathrm{Ae}_{21}\mathrm{A}, \Delta(\mathrm{e}_{22}) = \mathrm{Ae}_{22}\mathrm{A}.$$

Isbot. To`rtala $e_{ij} \in \mathrm{M}_2(\mathrm{R})$, $\mathrm{i}, \mathrm{j} = 1{,}2$ birlik matritsalarni juft-jufti bilan olsak, oltita juftlik hosil bo`ladi. Ushbu juftliklarning Δ ikki lokal ikki tomonlama ko`paytirishdagi ko`paytma matritsalarini belgilab olaylik.



U holda quyidagi tengliklar o`rinli bo`ladi.

$$\Delta(\mathrm{e}_{11}) = \mathrm{Be}_{11}\mathrm{B} = \mathrm{Ce}_{11}\mathrm{C} = \mathrm{Ne}_{11}\mathrm{N}, \Delta(\mathrm{e}_{12}) = \mathrm{Be}_{12}\mathrm{B} = \mathrm{De}_{12}\mathrm{D} = \mathrm{Me}_{12}\mathrm{M},$$
$$\Delta(\mathrm{e}_{21}) = \mathrm{Ce}_{21}\mathrm{C} = \mathrm{Ge}_{21}\mathrm{G} = \mathrm{Me}_{21}\mathrm{M}, \Delta(\mathrm{e}_{22}) = \mathrm{Ge}_{22}\mathrm{G} = \mathrm{De}_{22}\mathrm{D} = \mathrm{Ne}_{22}\mathrm{N}.$$

Birlik matritsalar uchun ikki lokal ikki tomonlama ko`paytirishni bittadan matritsa uchun hisoblab chiqamiz.

$$\Delta(\mathrm{e}_{11}) = \mathrm{Be}_{11}\mathrm{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} b_{11}^2 & b_{11}b_{12} \\ b_{21}b_{11} & b_{21}b_{11} \end{pmatrix}$$
$$\Delta(\mathrm{e}_{12}) = \mathrm{Be}_{12}\mathrm{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} b_{11}b_{21} & b_{11}b_{22} \\ {b_{21}}^2 & b_{21}b_{22} \end{pmatrix}$$
$$\Delta(\mathrm{e}_{21}) = \mathrm{Ce}_{12}\mathrm{C} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} c_{11}c_{12} & {c_{12}}^2 \\ c_{11}c_{22} & c_{22}c_{12} \end{pmatrix}$$
$$\Delta(\mathrm{e}_{22}) = \mathrm{Ge}_{22}\mathrm{G} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} g_{12}g_{21} & g_{12}g_{22} \\ g_{21}g_{22} & {g_{22}}^2 \end{pmatrix}$$

Ikki lokal ikki tomonlama ko`paytirishni bajarayotganda qolgan ikki matritsalarga ko`paytirib hisoblaganimizda ham, ko`paytmadagi matritsa elementlarining indekslari bir xil bo`ladi. Shu sababli

$$\mathrm{Be}_{11}\mathrm{B} = \mathrm{Ce}_{11}\mathrm{C} = \mathrm{Ne}_{11}\mathrm{N}, \mathrm{Be}_{12}\mathrm{B} = \mathrm{De}_{12}\mathrm{D} = \mathrm{Me}_{12}\mathrm{M},$$
$$\mathrm{Ce}_{21}\mathrm{C} = \mathrm{Ge}_{21}\mathrm{G} = \mathrm{Me}_{21}\mathrm{M}, \mathrm{Ge}_{22}\mathrm{G} = \mathrm{De}_{22}\mathrm{D} = \mathrm{Ne}_{22}\mathrm{N}$$

tenglikdan ularning barcha elementlarini tenglab chiqamiz:

$$b_{11}^2 = c_{11}^2 = n_{11}^2, b_{11}b_{12} = c_{11}c_{12} = n_{11}n_{12},$$

$$b_{11}b_{21} = c_{11}c_{21} = n_{11}n_{21}, b_{12}b_{21} = c_{12}c_{21} = n_{12}n_{21},$$
$$b_{11}b_{21} = d_{11}d_{21} = m_{11}m_{21}, b_{21}^2 = d_{21}^2 = m_{21}^2,$$
$$b_{21}b_{22} = d_{21}d_{22} = m_{21}m_{22}, c_{11}c_{12} = g_{11}g_{12} = m_{11}m_{12},$$
$$c_{12}^2 = g_{12}^2 = m_{12}^2, c_{11}c_{22} = g_{11}g_{22} = m_{11}m_{22},$$
$$c_{12}c_{22} = g_{12}g_{22} = m_{12}m_{22}, g_{22}g_{21} = d_{12}d_{21} = n_{12}n_{21},$$
$$g_{12}g_{22} = d_{12}d_{22} = n_{12}n_{22}, g_{22}g_{21} = d_{22}d_{21} = n_{22}n_{21},$$
$$g_{22}^2 = d_{12}^2 = n_{12}^2.$$

Qaralayotgan matritsalalr faqatgina noldan farqli musbat elementlardan tashkil topgan deb qaraymiz. U holda kvadrat tengliklarning elementlari o`zaro teng bo`ladi. Ya'ni:

$$b_{11} = c_{11} = n_{11}, b_{21} = d_{21} = m_{21}$$
$$c_{12} = g_{12} = m_{12}, g_{22} = d_{22} = n_{22}.$$

Bu tengliklarni yuqoridagi tengliklarga qo`llaganimizda, quyidagiga ega bo`lamiz:

$$b_{11} = c_{11} = d_{11} = g_{11} = n_{11} = m_{11}, b_{12} = c_{12} = d_{12} = g_{12} = n_{12} = m_{12},$$
$$b_{21} = c_{21} = d_{21} = g_{21} = n_{21} = m_{21}, b_{22} = c_{22} = d_{22} = g_{22} = n_{22} = m_{22}$$

Bundan B=C=D=G=N=M. Demak, barcha matritsalar o`zaro tengligiga erishdik. Isbot yakunlandi.

Teorema 1. Aytaylik Δ bu $M_2(R)$ ikki o`lchovli matritsalar algebrasida barcha komponentalari musbat bo`lgan, ya'ni $A = \{a_{ij}, a_{ij} > 0\}$ bo`lgan, matritsalar yordamida aniqlangan ikki lokal ikki tomonlama ko`paytirish bo`lsin. U holda shunday $A \in M_2(R)$ matritsa mavjudki, bunda ixtiyoriy $X \in M_2(R)$ matritsa uchun $\Delta(X) = AXA$ tenglik bajariladi, $\Delta$ ikki tomonlama ko`paytirish bo`ladi.

Isbot. Ixtiyoriy $x \epsilon M_2(R)$ uchun

$$\Delta(x) = BxB, \Delta(e_{11}) = Be_{11}B, \Delta(x) = CxC, \Delta(e_{12}) = Ce_{12}C$$
$$\Delta(x) = DxD, \Delta(e_{21}) = De_{21}D, \Delta(x) = FxF, \Delta(e_{22}) = Fe_{22}F.$$

1-Lemmaga ko`ra, shunday A mavjudki,

$$Be_{11}B = Ae_{11}A, Ce_{12}C = Ae_{12}A, De_{21}D = Ae_{21}A, Fe_{22}F = Ae_{22}A$$

tengliklar o`rinli bo`ladi. Bundan quyidagilar kelib chiqadi:

$$\begin{cases} b_{11}^2 = a_{11}^2 \\ b_{11}b_{12} = a_{11}a_{12} \\ b_{11}b_{21} = a_{11}a_{21} \\ b_{21}b_{12} = a_{21}a_{12} \end{cases} \begin{cases} c_{11}c_{21} = a_{11}a_{21} \\ c_{11}c_{22} = a_{11}a_{22} \\ c_{21}^2 = a_{21}^2 \\ c_{21}c_{22} = a_{21}a_{22} \end{cases}$$
$$\begin{cases} d_{11}d_{12} = a_{11}a_{12} \\ d_{12}^2 = a_{12}^2 \\ d_{11}d_{22} = a_{11}a_{22} \\ d_{22}d_{12} = a_{22}a_{12} \end{cases} \begin{cases} f_{21}f_{12} = a_{21}a_{12} \\ f_{12}f_{22} = a_{12}a_{22} \\ f_{21}f_{22} = a_{21}a_{22} \\ f_{22}^2 = a_{22}^2 \end{cases}$$

ga ega bo`lamiz. Bundan

$$a_{11} = b_{11} = c_{11} = d_{11}, a_{12} = b_{12} = f_{12} = d_{12},$$
$$a_{21} = b_{21} = c_{21} = f_{21}, a_{22} = c_{22} = d_{22} = f_{22}$$

Qulaylik uchun quyidagi belgilashlarni kiritib olamiz:

$$\alpha_{11} = a_{11}^2x_{11} + a_{11}a_{12}x_{21} + a_{11}a_{21}x_{12} + a_{12}a_{21}x_{22},$$
$$\alpha_{12} = a_{11}a_{12}x_{11} + a_{12}^2x_{21} + a_{11}a_{22}x_{12} + a_{12}a_{22}x_{22},$$
$$\alpha_{21} = a_{21}a_{11}x_{11} + a_{11}a_{22}x_{21} + a_{21}^2x_{12} + a_{21}a_{22}x_{22},$$
$$\alpha_{22} = a_{21}a_{12}x_{11} + a_{12}a_{22}x_{21} + a_{11}a_{22}x_{21} + a_{22}^2x_{22},$$
$$\beta_{11} = b_{11}^2x_{11} + b_{11}b_{12}x_{21} + b_{11}b_{21}x_{12} + b_{12}b_{21}x_{22},$$
$$\gamma_{21} = c_{21}c_{11}x_{11} + c_{11}c_{22}x_{21} + c_{21}^2x_{12} + c_{21}c_{22}x_{22},$$

$$\delta_{12} = d_{11}d_{12}x_{11} + d_{12}^2x_{21} + d_{11}d_{22}x_{12} + d_{12}d_{22}x_{22},$$
$$\varepsilon_{22} = f_{21}f_{12}x_{11} + f_{12}f_{22}x_{21} + f_{11}f_{22}x_{21} + f_{22}^2x_{22}.$$

U holda

$$\Delta(x) = BXB = \begin{pmatrix} \beta_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}, \Delta(x) = CXC = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \gamma_{21} & \alpha_{22} \end{pmatrix},$$
$$\Delta(x) = DXD = \begin{pmatrix} \alpha_{11} & \delta_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}, \Delta(x) = FXF = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \varepsilon_{22} \end{pmatrix}.$$

Demak,

$$\Delta(x) = BXB = CXC = DXD = FXF = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}.$$

Shunday $\mathrm{A} \in \mathrm{M}_2(\mathrm{R})$ borki, ixtiyoriy $\mathrm{x} \in \mathrm{M}_2(\mathrm{R})$ olinganda, $\Delta(\mathrm{X}) = \mathrm{AXA}$ tenglik o`rinli bo`ladi. Teorema isbotlandi.

Aytaylik, $M_2(R) \otimes C[a, b]$ bu komponentalari $C[a, b]$ uzluksiz funksiyalar algebrasidan olingan barcha 2×2 o`lchovli matritsalar algebrasi bo`lsin.

Lemma 3. Aytaylik, Δ bu $M_2(R) \otimes C[a, b]$ ikki o`lchovli matritsalar algebrasida barcha komponentalari musbat funksiya bo`lgan matritsalar yordamida aniqlangan ikki lokal ikki tomonlama ko`paytirish bo`lsin. U holda ikki o`lchovli matritsalar algebrasida shunday $\mathrm{A} \in M_2(R) \otimes C[a, b]$ matritsa mavjudki, barcha birlik $e_{ij}(\mathrm{t}) \in M_2(R) \otimes C[a, b]$, $\mathrm{i}, \mathrm{j} = 1{,}2$ matritsalar uchun $\Delta$ikki lokal ikki tomonlama ko`paytirishda $\Delta(\mathrm{e_{ij}}) = \mathrm{Ae_{ij}A}$ tenglik bajariladi, ya'ni:

$$\Delta(\mathrm{e}_{11}(\mathrm{t})) = \mathrm{Ae}_{11}(\mathrm{t})\mathrm{A},\ \Delta(\mathrm{e}_{12}(\mathrm{t})) = \mathrm{Ae}_{12}(\mathrm{t})\mathrm{A}$$
$$\Delta(\mathrm{e}_{21}(\mathrm{t})) = \mathrm{Ae}_{21}(\mathrm{t})\mathrm{A}, \Delta(\mathrm{e}_{22}(\mathrm{t})) = \mathrm{Ae}_{22}(\mathrm{t})\mathrm{A}.$$

Teorema 4. Aytaylik, Δ bu $M_2(R) \otimes C[a, b]$ ikki o`lchovli matritsalar algebrasida barcha komponentalari musbat funksiya bo`lgan matritsalar yordamida aniqlangan ikki lokal ikki tomonlama ko`paytirish bo`lsin, ya'ni $\Delta: M_2(R) \otimes C[a, b] \ \to M_2(R) \otimes C[a, b]$ shunday ikki lokal ikki tomonlama ko`paytirish bo`lsinki ixtiyoriy $x, y \,\epsilon\, M_2(R) \otimes C[a, b]$ elementlar uchun, $\Delta(x) = axa\ \Delta(y) = aya$ shartni qanoatlantiruvchi $a = \begin{pmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{pmatrix}$ matritsa uchun, ixtiyoriy $x\epsilon[a, b]$ uchun $a_{11}(t) > 0$, $a_{12}(t) > 0, a_{21}(t) > 0, a_{22}(t) > 0$ bo`lsin. U holda $\Delta$ ikki tomonlama ko`paytirish bo`ladi, ya'ni shunday $a\epsilon M_2(R) \otimes C[a, b]$ topiladiki, ixtiyoriy $x\epsilon\ M_2(R) \otimes C[a, b]$ uchun $\Delta(x) = axa$ o`rinli bo`ladi.

Isbot. Ixtiyoriy $x\epsilon M_2(R) \otimes C[a, b]$ uchun

$$\Delta(\mathrm{x}) = \mathrm{BxB}, \Delta(\mathrm{e}_{11}(t)) = \mathrm{Be}_{11}(t)\mathrm{B}$$
$$\Delta(\mathrm{x}) = \mathrm{CxC}, \Delta(\mathrm{e}_{12}(t)) = \mathrm{Ce}_{12}(t)\mathrm{C}$$
$$\Delta(\mathrm{x}) = \mathrm{DxD}, \Delta(\mathrm{e}_{21}(t)) = \mathrm{De}_{21}(t)\mathrm{D}$$
$$\Delta(\mathrm{x}) = \mathrm{FxF}, \Delta(\mathrm{e}_{22}(t)) = \mathrm{Fe}_{22}(t)\mathrm{F}.$$

3-Lemmaga ko`ra, shunday A mavjudki,

$$\mathrm{Be}_{11}(t)\mathrm{B} = \mathrm{A}e_{11}(t)A, \mathrm{Ce}_{12}(t)\mathrm{C} = \mathrm{A}e_{12}(t)A,$$
$$\mathrm{De}_{21}(t)\mathrm{D} = \mathrm{A}e_{21}(t)A, \mathrm{Fe}_{22}(t)\mathrm{F} = \mathrm{A}e_{22}(t)A$$

tenglik o`rinli, ya'ni:

$$\begin{cases} b_{11}^2(t) = a_{11}^2(t) \\ b_{11}(t)b_{12}(t) = a_{11}(t)a_{12}(t) \\ b_{11}(t)b_{21}(t) = a_{11}(t)a_{21}(t) \\ b_{21}(t)b_{12}(t) = a_{21}(t)a_{12}(t) \end{cases}$$

$$\begin{cases} c_{11}(t)c_{21}(t) = a_{11}(t)a_{21}(t) \\ c_{11}(t)c_{22}(t) = a_{11}(t)a_{22}(t) \\ c_{21}^2(t) = a_{21}^2(t) \\ c_{21}(t)c_{22}(t) = a_{21}(t)a_{22}(t) \end{cases}$$

$$\begin{cases} d_{11}(t)d_{12}(t) = a_{11}(t)a_{12}(t) \\ d_{12}^2(t) = a_{12}^2(t) \\ d_{11}(t)d_{22}(t) = a_{11}(t)a_{22}(t) \\ d_{22}(t)d_{12}(t) = a_{22}(t)a_{12}(t) \end{cases}$$

$$\begin{cases} f_{21}(t)f_{12}(t) = a_{21}(t)a_{12}(t) \\ f_{12}(t)f_{22}(t) = a_{12}(t)a_{22}(t) \\ f_{21}(t)f_{22}(t) = a_{21}(t)a_{22}(t) \\ f_{22}^2(t) = a_{22}^2(t) \end{cases}$$

ga ega bo`lamiz. Bundan

$$a_{11}(t) = \mathrm{b}_{11}(\mathrm{t}) = \mathrm{c}_{11}(\mathrm{t}) = \mathrm{d}_{11}(\mathrm{t}), a_{12}(t) = \mathrm{b}_{12}(\mathrm{t}) = \mathrm{f}_{12}(\mathrm{t}) = \mathrm{d}_{12}(\mathrm{t}),$$
$$a_{21}(t) = \mathrm{b}_{21}(t) = \mathrm{c}_{21}(t) = \mathrm{f}_{21}(t), a_{22}(t) = \mathrm{c}_{22}(t) = \mathrm{d}_{22}(t) = \mathrm{f}_{22}(t)$$

Qulaylik uchun quyidagi belgilashlarni kiritib olamiz:

$$\alpha_{11}(t) = a_{11}^2(t)x_{11} + a_{11}(t)a_{12}(t)x_{21} + a_{11}(t)a_{21}(t)x_{12} + a_{12}(t)a_{21}(t)x_{22},$$
$$\alpha_{12} = a_{11}(t)a_{12}(t)x_{11} + a_{12}^2(t)x_{21} + a_{11}(t)a_{22}(t)x_{12} + a_{12}(t)a_{22}(t)x_{22},$$
$$\alpha_{21}(t) = a_{21}(t)a_{11}(t)x_{11} + a_{11}(t)a_{22}(t)x_{21} + a_{21}^2(t)x_{12} + a_{21}(t)a_{22}(t)x_{22},$$
$$\alpha_{22}(t) = a_{21}(t)a_{12}(t)x_{11} + a_{12}(t)a_{22}(t)x_{21} + a_{11}(t)a_{22}(t)x_{21} + a_{22}^2(t)x_{22},$$
$$\beta_{11}(t) = b_{11}^2(t)x_{11} + b_{11}(t)b_{12}(t)x_{21} + b_{11}(t)b_{21}(t)x_{12} + b_{12}(t)b_{21}(t)x_{22},$$
$$\gamma_{21}(t) = c_{21}(t)c_{11}(t)x_{11} + c_{11}(t)c_{22}(t)x_{21} + c_{21}^2(t)x_{12} + c_{21}(t)c_{22}(t)x_{22},$$
$$\delta_{12}(t) = d_{11}(t)d_{12}(t)x_{11} + d_{12}^2(t)x_{21} + d_{11}(t)d_{22}(t)x_{12} + d_{12}(t)d_{22}(t)x_{22},$$
$$\varepsilon_{22}(t) = f_{21}(t)f_{12}(t)x_{11} + f_{12}(t)f_{22}(t)x_{21} + f_{11}(t)f_{22}(t)x_{21} + f_{22}^2(t)x_{22}.$$

U holda

$$\Delta(x) = BXB = \begin{pmatrix} \beta_{11}(t) & \alpha_{12}(t) \\ \alpha_{21}(t) & \alpha_{22}(t) \end{pmatrix}, \Delta(x) = CXC = \begin{pmatrix} \alpha_{11}(t) & \alpha_{12}(t) \\ \gamma_{21}(t) & \alpha_{22}(t) \end{pmatrix},$$
$$\Delta(x) = DXD = \begin{pmatrix} \alpha_{11}(t) & \delta_{12}(t) \\ \alpha_{21}(t) & \alpha_{22}(t) \end{pmatrix}, \Delta(x) = FXF = \begin{pmatrix} \alpha_{11}(t) & \alpha_{12}(t) \\ \alpha_{21}(t) & \varepsilon_{22}(t) \end{pmatrix}.$$

Demak,

$$\Delta(x) = BXB = CXC = DXD = FXF = \begin{pmatrix} \alpha_{11}(t) & \alpha_{12}(t) \\ \alpha_{21}(t) & \alpha_{22}(t) \end{pmatrix}.$$

Teorema isbotlandi.

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DESCRIPTION OF 2-LOCAL TWO SIDED MULTIPLICATION ON AN ALGEBRA OF MATRICES

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The present paper is devoted to 2-local derivation on associative and Jordan matrix rings. In 1997, P.Semrl introduced the notion of 2-local derivations and described 2-local derivations on the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space H. A similar description for the finite-dimensional case appeared later in 2004. In the paper Y. Lin and T. Wong 2-local derivations have been described on matrix algebras over finite dimensional division rings. In 2012 Sh. Ayupov, K. Kudaybergenov suggested a new technique and generalized the above mentioned results for abritrary Hilbert spaces. Namely they considered 2-local derivations on the algebra $B(H)$ of all linear bounded operators on an arbitrary (no separability is assumed) Hilbert space H and proved that every 2-local derivation on $B(H)$ is a derivation. Later Sh.Ayupov, K.Kudaybergenov and F.Arzikulov extended the above results and gave a proof of the theorem for an arbitrary von Neumann algebra.

In the given paper it is introduced and studied a concept of 2-local two sided multuplication on $\mathrm{M}_2(\mathrm{R})$. A 2-local two sided multuplication is defined as follows:let $\Delta\colon \mathrm{M}_2\mathrm{R} \to \mathrm{M}_2\mathrm{R}$ be a mapping. If for each pairs $\mathrm{X}, \mathrm{Y} \in \mathrm{M}_2(\mathrm{R})$ of elements there exists a matrix $\mathrm{A} \in \mathrm{M}_2(\mathrm{R})$ such that $\Delta(\mathrm{x}) = \mathrm{AXA}$, $\Delta(\mathrm{y}) = \mathrm{AYA}$, then the mapping Δ is called 2-local two-sided multuplication. It is proved that every 2-local two sided multuplication of $\mathrm{M}_2(\mathrm{R})$ is a two sided multuplication, if all components of all matrices defining this 2-local two sided multuplication are positive. Namely, we prove that for any matrix $x \in \mathrm{M}_2(\mathrm{R})$ there exists a matrix $a \in \mathrm{M}_2(\mathrm{R})$ with positive entries such that $\varphi(x) = axa$. We prove the following lemma: let Δ be a 2-local two sided multuplication on $\mathrm{M}_2(\mathrm{R})$ defined by 2×2 matrices with posotive entries of the field of real numbers. Then there exists a matrix $\mathrm{A} \in \mathrm{M}_2(\mathrm{R})$ with positive entries such that, for all matrix units $e_{ij} \in \mathrm{M}_2(\mathrm{R})$, $\mathrm{i}, \mathrm{j} = 1,2$ it is valid $\Delta(\mathrm{e_{ij}}) = \mathrm{Ae_{ij}A}$ for the 2-local two sided multuplication Δ, that is $\Delta(\mathrm{e}_{11}) = \mathrm{Ae}_{11}\mathrm{A}$, $\Delta(\mathrm{e}_{12}) = \mathrm{Ae}_{12}\mathrm{A}$, $\Delta(\mathrm{e}_{21}) = \mathrm{Ae}_{21}\mathrm{A}$, $\Delta(\mathrm{e}_{22}) = \mathrm{Ae}_{22}\mathrm{A}$.

Similarly we define a concept of 2-local two sided multuplication on $\mathrm{M}_2(\mathrm{R}) \otimes C[a,b]$ as follows: let $\Delta\colon \mathrm{M}_2\mathrm{R} \otimes C[a,b] \to \mathrm{M}_2\mathrm{R} \otimes C[a,b]$ be a mapping. If for each pairs $\mathrm{X}, \mathrm{Y} \in \mathrm{M}_2(\mathrm{R}) \otimes C[a,b]$ of elements there exists a matrix $\mathrm{A} \in \mathrm{M}_2(\mathrm{R}) \otimes C[a,b]$ such that $\Delta(\mathrm{x}) = \mathrm{AXA}$, $\Delta(\mathrm{y}) = \mathrm{AYA}$, then the mapping Δ is called 2-local two-sided multiplication. In the present paper, it is proved that every 2-local two sided multuplication of $M_2(R) \otimes C[a,b]$ is a two sided multuplication, if all components of all matrices defining this 2-local two sided multuplication are positive functions. Namely, we prove that for any matrix $x \in M_2(R) \otimes C[a,b]$ there exists a matrix $a \in \mathrm{M}_2(\mathrm{R})$ with positive function entries such that $\varphi(x) = axa$.

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